High School

MATH & SCIENCE LESSON 6

Hilleman & Vaccines

LEARNING GOALS

This series of lessons will allow students to use mathematics to improve scientific and mathematical literacy, and combine the two to help students understand where humans are in the context of a pandemic, especially during the development of an entirely new vaccine. Students will understand the development and use of a variety of vaccines.

WHERE DOES THIS FIT INTO YOUR CURRICULUM?

MATH

Using simulations as models and then applying computational thinking to understand processes

Using probability and statistics to understand population dynamics

SCIENCE

Understanding the spread of diseases in populations

Understanding the development and use of a variety of vaccines and the process of achieving herd immunity to stabilize the human population during a pandemic

Understanding how genetic mutations occur and their effect on organisms and the stability of a system

MATHEMATICAL PRACTICES

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

MATHEMATICAL STANDARDS

- Statistics and Probability: Interpreting Categorical and Quantitative Data (S.ID1,2,3, 5, 6a,6b, 6c).
- Conditional Probability and the Rules of Probability (S.CP 1,2,3,4,5,6,7,8).
- Making Inferences and Justifying Conclusions (S.IC, 1,2,3,4,5,6).
- Using Probability to Make Decisions: (S.MD 6,7).

NEXT GENERATION SCIENCE STANDARDS ALIGNMENT

***HS-LS2-1*: Interdependent Relationships in Ecosystems**

Use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.

***HS-LS3-2* Inheritance and Variation of Traits**

Make and defend a claim based on evidence that inheritable genetic variations may result from (1) new genetic combinations through meiosis, (2) viable errors occurring during replication, and/or (3) mutations caused by environmental factors.

***HS-LS3-3* Inheritance and Variation of Traits**

Apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population.

MONTANA SCIENCE STANDARDS

Crosscutting Concepts: Cause and effect; proportion and quantity, and systems and system models.

Science and Engineering Practices: Developing and using models; analyzing and interpreting data; using mathematics and computational thinking, constructing explanations as it applies to science.

LS2. A: Use mathematical or computational representations to support arguments about environmental factors that affect carrying capacity, biodiversity, and populations in ecosystems.

LS3.B: Make and defend a claim based on evidence from multiple sources that inheritable genetic variation may result from:

- New genetic combinations through meiosis
- Viable errors occurring during replication
- Mutations caused by environmental factors

LS3. B: Apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population.

Lesson 6

Probability (Recommended 11-12 grades)

Hilleman & Vaccines

LESSON INSTRUCTIONS
Lesson 6: Probability

INTRODUCTION

Have students share the graphs they made with the class. Use the provided grading rubric to score the graphs.

ENGAGEMENT

Students will use the vaccine efficacy data from Lesson 1 to learn about probability.

OBJECTIVE

Students will learn to use permutations and combinations to compute probabilities of compound events and solve problems.

INSTRUCTIONS

Begin by introducing binomial experiments when probability is not equal to 1/2:

Binomial Experiment Requirements:

- There is a fixed number of trials.
- There are two mutually exclusive outcomes for each trial: success and failure. P(success) + P(failure) = 1.
- The trials are independent and have the same P(success).

Discussion here is appropriate about how the vaccine trials fit this model.

Formula used: (nCr)(S^r)(F^(n-r))

Where nCr is used to determine the number of ways to select exactly r subjects from a total of n subjects, “S” is the probability of the favorable outcome , and F is the probability of the unfavorable outcome.

For example, when rolling a number cube the probability of rolling a "2" is ⅙, and the probability of not rolling a "2" is ⅚. To find the probability that when rolling a fair number cube 5 times, exactly 4 will display a "2", use the formula in this way:
5C4(⅙)^4(⅚)^1. Answer: 0.0032 or 0.32 %

Replacing the word "exactly" in this problem with "at least" changes this problem to :

Find the probability that when rolling a fair number cube 5 times, at least 4 will display a "2".

In symbolic notation this problem changed from:
P(exactly 4 2's in 5 rolls) to P(exactly 4 2's in 5 rolls OR exactly 5 2's in 5 rolls). The calculations would then be carried out in this way:
5C3(⅙)^3(⅚)^2 + 5C4(⅙)^4(⅚)^1. Answer: 0.0322 + 0.0032 = 0.0354 or 3.54%

ASSESSMENT

Students will answer the questions on student pages 208-209.

Student Worksheet - ANSWER KEY

Given the success rates for some vaccines, suppose 4 children under the age of ten have each had the minimum doses of all of the vaccines. (MMR, Polio, DPT, HB, HIB).

Given:
Percent Efficacy (Success Rates):
Source: https://www.cdc.gov/mmwr/preview/mmwrhtml/00048610.htm

Measles (Merck Sharp and Dohme Corp., 2018): 98.8
Mumps (Merck Sharp and Dohme Corp., 2018): 99.5
Rubella (Merck Sharp and Dohme Corp., 2018): 99.5
Pertussis (MMWR, 1997): DTaP: 81%
Polio (MMWR, 1997): 100
Hepatitis B (MMWR, 1997): 95%

Find the Probabilities for Each Situation

1. All four children are exposed to measles.
   All four children get the disease.
   Answer: 4C4 (0.012^4)(0.988^0)= 2.0736 x 10^{-8} = 0.000000020736

2. All four children are exposed to polio.
   None of the children gets the disease.
   Answer: 1 (Students may try to enter this into the formula and discover that the technology responds with a "domain error", which happens when rotely following a formula and entering 0^0.)

3. All four children are exposed to mumps.
   Only one child gets the disease.
   Answer: 4C1(0.005^1)(0.995^3)=0.0197

4. All four children are exposed to pertussis.
   At least three children get the disease.
   Answer: 4C4(0.19^4)(.81^0) + 4C3(.19^3)(.81^1)= .0013 + 0.222 - 0.0235 = 2.35%

Student Worksheet - ANSWER KEY

For 5-8, suppose 10 children under the age of ten have each had the minimum doses of all of the vaccines. (MMR, Polio, DPaT, HB, HIB). *(Teacher may consider changing this number exposed to the number in your classroom)*

5. All 10 children are exposed to rubella. 2 children get the disease.
   Answer: 10C2(0.005^2)(0.995^8) = 0.0010807797 = 0.108%

6. All 10 children are exposed to rubella. None of them get the disease.
   Answer: 10C0(0.005^0)(0.995^10) = 0.9511 = 95.11%

7. All 10 children are exposed to pertussis. 2 children get the disease.
   Answer: 10C2(0.19^2)(0.81^8) = 0.301 = 30.1%

8. All 10 children are exposed to Hepatitis B. 3 children get the disease.
   Answer: 10C3(.05^3)(.95^7) = 0.01048 = 1.048%

Student Worksheet

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Find the Probabilities for Each Situation

1. All four children are exposed to measles.
   All four children get the disease.

2. All four children are exposed to polio.
   None of the children gets the disease.

3. All four children are exposed to mumps.
   Only one child gets the disease.

4. All four children are exposed to pertussis.
   At least three children get the disease.

Student Worksheet

For 5-8, suppose 10 children under the age of ten have each had the minimum doses of all of the vaccines. (MMR, Polio, DPaT, HB, HIB).

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8. All 10 children are exposed to Hepatitis B. 3 children get the disease.

Bibliography

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